SERIAL AND PARALLEL SOLUTIONS TO VARIANTS OF VEHICLE ROUTING PROBLEM USING SWARM INTELLIGENCE TECHNIQUES

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1. Introduction

Distribution management systems have become increasingly complex during recent years. A wide range of organizations and companies that deals with transportation of goods or people is facing this problem. This is because constraints and objectives encountered in practice are highly variable and varies for each setting. Since the generalized Travelling Salesman Problem (TSP) (Dantzig & Ramser, 1959) was introduced; cost has been the main constraint to focus. But problem is now more complex due to many other factors such as vehicle count and delivery time (Toth & Vigo, 2001) etc. Therefore the interest in Vehicle Routing Problem (VRP) has developed from a small group of mathematicians to the broad range of practitioners and researchers.

VRP is an NP hard combinatorial optimization problem that has exact solution only for small instances of the problem. Although, a number of techniques have been introduced for solving the VRPs but they do not guarantee optimality. In the last two-three decades the meta-heuristics approaches have emerged as the most promising solution for the VRP problems. Also parallel computing methods are increasingly being proposed to effectively solve the large and complex optimization problems. Dates back it takes weeks or months to find a solution to large and complex problems, but now due to parallelization the running time cut down to days or even hours which make to run several tests in a short time. Similarly, the multi-objective optimization is attracting many researchers' attention, because of its ability to optimize two or more conflicting objectives simultaneously.

1.1 Combinatorial Optimization Problem

These are the problems in which, a set of feasible solutions is discrete or it can be reduced to make discrete, so that a best optimal solution can be obtained that minimizes the given function. Branch and Bound or Branch and Cut are some methods to solve these problems, but not large and complex problems. As the problem complexity increases, the algorithm's cost also increases and optimality of the solution reduces, which makes the solution non-feasible. Hence the other way to optimize these problems is to find a suboptimal solution in a reasonable time by using Heuristics & Meta-heuristics techniques.

1.2 Vehicle Routing Problem (VRP) and its Variants

VRP is one of the most important logistic problem in the domain of distribution and transportation, concerned to find a *minimum-cost set of routes* chosen by different vehicles for distribution of goods, while satisfying certain constraints.

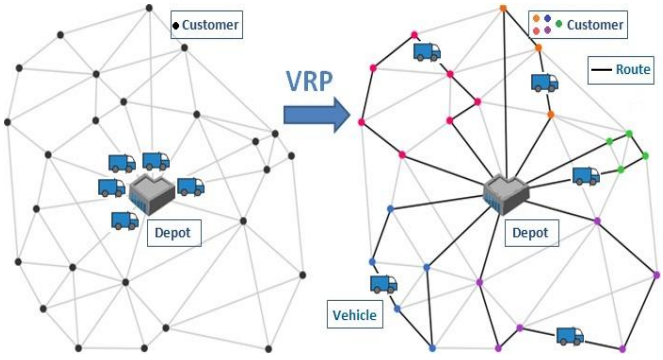

Figure 1 : Vehicle Routing Problem

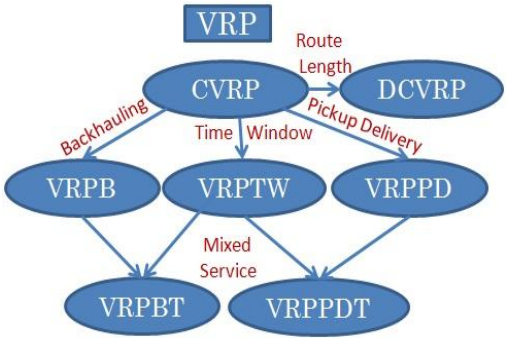

Figure 2 : Different Versions of VRP

It holds a great number of real life applications, such as transportation of people and goods, delivery services, sales people routing, garbage collection, school bus routing and dial-a-ride systems (Toth & Vigo, 2001). It can be applied for vehicles, planes, trains; that is why it has great importance in real life. VRP has many objectives and several constraints to be considered, which actually calls the different versions of VRP that have been introduced in the last few decades. Next sections, will define the different versions of VRP, which have their own characteristics.

Capacitated Vehicle Routing Problem (CVRP)

CVRP is the elemental variant of VRP, and can be defined as a complete graph $G = (V, A)$, in which $V = \{0,...,n\}$ is the vertex set and 'A' is the arc set. Vertices $1,...,n$ represent the clients, having a known and deterministic demand 'q_i', and $V(0)$ corresponds to the depot. A travel cost associated to each arc belongs to 'A' between nodes i & j is denoted by C_{ij} (Toth & Vigo, 2001). CVRP takes into account usually a homogeneous fleet of 'm' identical vehicles with restricted capacity (in terms of maximum weight, volume, or number of pallets, which they cannot exceed). The CVRP calls out for the determination to find the minimum travel cost of a set of 'K' routes, when:

[1] Each client $V\{0\}$ should be visited exactly once,

[2] Each vehicle route originates and terminates at the same depot

[3] The sum of demands of clients should not exceed the vehicle capacity 'Q'.

A solution of this type of VRP can be seen as a set of 'K' cycles (like in figure 1) sharing a common depot.

One universal assumption made for all the VRP variants (according to previous literature) is that every vehicle belongs to a particular depot and has to start and end its tour at that same depot.

Distance - Constrained Vehicle Routing Problem (DVRP)

The second variant we are considering is the *DVRP*, in which maximum length is the main constraint instead of capacity. In particular, a non-negative length, l_{ij} or *le* is now associated with each arc and it is important that for each route, the total length of the arcs should be less than the maximum tour (route) length, *L*. Hence, the aim in the DVRP is to *minimize the total route length*. The case with both capacity and distance constraints is named as *Distance-Constrained VRP* (DCVRP).

Vehicle Routing with Time Windows (VRPTW)

VRPTW is an important version of VRP that has been attracted many researchers within the last 10 - 12 years. The VRPTW additionally has restrictions on the service time i.e. a time interval called as time window is associated to each client. Therefore, the time instant for a route includes: (i) the time in which vehicle departs from depot, (ii) the total travel time, and (iii) the service time for each client (Toth & Vigo, 2001). The service for each client must begin within the allotted time interval and for that time vehicle must stop at the customer location. Moreover, vehicle will wait, if it reaches before the service time starts. Also, the time window constraint induces an indirect orientation to each tour even if they are symmetric. Therefore, the VRPTW usually modelled as an asymmetric problem. VRPTW calls to find a minimum cost set of 'K' simple circuits, such that:

[1] It follows the standard constraints of VRPs (universal assumption)

[2] Services for all customers shuold initiate within the time period (time window) and each vehicle should halt for some time instant 's_i' at each customer location.

Vehicle Routing Problem with Backhauls (VRPB)

In this *VRPB, c*ustomers are divided into two categories: (i) *Linehaul customers,* requiring a delivery of the given quantity of products and (ii) *Backhaul customers,* for which given quantity of products have to be picked up. In the VRPB, there exists a precedence constraint between both groups of customers, i.e., on every route, the linehaul customers must be served before the backhaul customers. A non-negative demand, 'd_t' (t, type of demand), to be delivered or collected, is associated with each customer. VRPB finds minimum cost set for '*K'* circuits, such that:

[1] Standard conditions of VRPs (given above) should be followed.

[2] The separate sum of the linehaul and backhaul customers' demands should be less than the vehicle capacity '*Q*'; and

[3] For each circuit the backhaul clients served after the linehaul clients.

The case of VRP backhauling with time windows (VRPBTW) has been also proposed in the past studies.

VRP variant with Pickup and Delivery (VRPPD)

In *VRPPD*, different vehicles, located at multiple points, fulfil a set of customers requests. It can be define by considering some variables i.e a point for pick up, a point for delivery and a demand to be satisfy between these points. In the basic VRPPD, a customer *'i'* has two properties *'d_i'* and *'p_i'* i.e. the required demand of the corresponding items to be delivered and to be picked up. Let *'O_i'* is the point of origin for delivery and *'D_i'* is the destination point for pickup. It is always assumed, that the delivery must be performed before the pickup in order to follow thw capacity constraint. Therefore, the load currently a vehicle has before arriving at next location can be calculated as: [initial load - demands that are delivered + the demands already picked up]. The VRPPD consists of finding minimum cost set as:

[1] It follows the standard conditions of VRPs (given above)

[2] Customers 'O_i', when different from depot, must be served in the same circuit before customer *'i'*;

[3] The customer *'D_i'*, when different from depot, must be served in the same circuit and after customer *'i'*.

Usually, starting and the terminating points of the demands are common (as in CVRP and VRPB), and the case is known as the *VRP with Simultaneous Pickup and Delivery (VRPSPD)*. The case of VRPPD with time windows is an another variant named as *VRPPDTW*.

Multiple Depot Vehicle Routing Problem (MDVRP)

A company may have several service points from where it serves its customers, hence it is a case of more than one depots and the problem is called *Multiple Depot VRP (MDVRP)*. Since there can be a large number of service points (depots) so it is a difficult to determine the particular service point for

customers service without violating the capacity constraint. Hence grouping is performed and customers are grouped (clustered) around depots according to the distance between customers and the depots. However, if the depots and the customers are intermingled then a Multi-Depot Problem can be solved.

The objective is to fulfill all customers' demands of commodities from several depots, while minimizing the vehicle count, the total travel time and the travel distance. A feasible solution can be found if each route follows the VRP constraints, also starts and terminates at the same depot. The minimum-cost of the route is calculated like as standard VRP.

Periodic Vehicle Routing Problem (PVRP)

PVRP is the case, where the basic VRP is generalized by extending the scheduling service period for *N* days. Thus, vehicles might not return in the same day it departs. Additionally, each customer may be visited more than once in this N-day period. The PVRP can be formulated as (Francis, Smilowitz, & Tzur, 2008): A Period set $P = \{\delta_1, \ldots, \delta_N\}$ of N days that represents the planning period. A schedule is a collection of days within the planning period in which customer services are fulfilled. Allocating a customer to a schedule indicates that the customer will receive services in every day of that schedule. Also, each customer has a fixed day to day demands that must be fulfilled by exactly one vehicle in only one visit. Hence, each customer may be visited more than one time in a particular schedule. Hence to minimize the global cost; a feasible route of an ordered sequence of service should satisfy the following constraints:

[1] Standard VRP constraints.

[2] The sequence of customers must be visited by the same vehicle during the day *'δn'*.

[3] Time to travel the customer sequence should not exceed the vehicle's daily service time.

Split Delivery Vehicle Routing Problem (SDVRP)

The *Split Delivery VRP (SDVRP)* was first introduced by Dror & Trudeaut (Dror, Laporte, & Trudeau, 1994), they suggested a way to generate savings by allowing split deliveries. In this context, it is allowed to serve the same client by different vehicles as long as the service plan reduces the overall cost. This relaxation is very important (Archetti & Speranza, 2008), where the customer orders sizes are as big as the capacity of the vehicle. A VRP can be transform into a SDVRP is by dividing each customer's order into small orders. It gives a feasible solution, by staisfying all VRP constraints, except a customer can be served by more than one vehicle. The aim is to find minimum of: (i) the total cost of all routes, (ii) the vehicle fleet (iii) the travelling time required to serve all customers.

Stochastic Vehicle Routing Problem (SVRP)

The *Stochastic VRP (SVRP)* by Hadjiconstantinou and Roberts (Hadjiconstantinou & Roberts, 2001), differs from the VRP, due to one or more random variables in the system. Common examples of stochastic elements are (Gendreau, Laporte, & Eguin, 1995):

[1] ***Stochastic customers***: Sometimes, the valid customers sets are not known. So, each customer has a probability of being present or absent i.e. p_i and $1 - p_i$ respectively.
[2] ***Stochastic demands***: For each customer, the demand 'd_i' is random.
[3] ***Stochastic travel times***: Service times 'δ_i' and travel times 't_{ij}' are random variables.

In SVRP, two ways are made to find a solution: (1) determine a solution before knowing the realizations of the random variables (2) take recourse or corrective action when the values of the random variables are known. When the data is random, it is not possible to satisfy all constraints for all realizations. So, the decision maker either satisfies some of the constraints with a given probability, or takes some corrective actions on violation of some constraints. For example; in the capacity constrained SVRP (with collections), vehicle may return to the depot for possible corrective actions: (i) when the vehicle is full, so returns to unload the collection, and then resume the service plan (ii) when the vehicle is full and return to re-optimize the remaining part of the planned route (iii) vehicle is not full, but it is known that going to the next customer could exceed its capacity. The aim of the problem is to minimize the vehicle count, the sum of routes taken and the sum of travel time required to service all customers with random values.

1.3 Multi-objective Optimization

Multi-objective is the process of optimizing two or more conflicting objectives simultaneously that is subject to certain constraints. It can be found in almost all realistic situations, where the solution obtained minimizes some objectives and maximizes other objectives at the same time. Single solution cannot simultaneously minimize and maximize each objective of a well formed multi-objective problem. Hence solution should be such that optimizes each objective to its extent. Finding such solutions and quantifying them by comparing to others is the main goal when solving a multi-objective optimization problem. Two general approaches of multiple objective optimizations methods are:

1. Scalar Methods (Preference-based Classical Approach)
2. Pareto optimal approach (Ideal Approach)

Scalar method also known as preference-based method (Tan, Chew, & Lee, 2006) solves the problem, by constructing an aggregate objective function which is aggregation of all the objectives. Here, weight of an objective is proportional to the preference vector assigned to that objective. This method converts the multi-objective problem to single-objective problem by scalarizing an objective vector to a single composite objective function. Hence, by optimizing such objective function, one particular trade-off solution is obtained. This method of solving multi-objective problems is simple but relatively subjective. It can be combined with heuristics or meta-heuristics for any single objective, according to the past research. For many multi-objective problems on VRP, *Scalar* method has been used with many heuristics (Blum & Andrea, 2003); genetic algorithms (Ombuki, Ross, & Hanshar, 2006), and local search algorithms (Murata & Itai, 2007).

The second approach known as *Pareto Optimal approach*, which determines a non-dominated set of solutions. While moving from one Pareto solution to another, there encounters a certain loss or gain in one or more objectives so that certain gain or loss can be achieved in other(s). Pareto optimal solution sets are usually implement on single solutions due of their practical applications in real-life problems. The size of Pareto sets is usually increased when the number of objective increases. In many multi-

objective (like routing) problems, the *Pareto* method is widely used (Doerner, Focke, & Gutjahr, 2007; Geiger, 2008) (Jozefowiez, Semet, & Talbi, 2004) (Tan, Chew, & Lee, 2006) to solve the VRPs.

Note that, the relative preference vector (preference-based) is highly subjective, therefore not so simple and easy to compute, whereas, the Pareto optimal approach is more practical and less subjective.

1.4 Heuristics and Meta-heuristics Techniques

The VRP problem has been extensively studied and discussed since its introduction and a large number of heuristics and meta-heuristics approaches have been proposed for solving the VRPs various probloems. This section will discuss the possible computational methods suggested in literature to solve the VRP problems.

Heuristics Approaches

Mainly proposed between the 1960s and 1980s, an wide range of heuristics proposed for constructive solutions to the VRP. These approaches are easy to perform and can find better solutions to large and complex combinatorial problems with relatively less computational effort; however, fail to find the global optimum solution. The solution obtained for the problem may not be the best among all actual solutions, or it may be simply approximate to the exact solution. But it is still valuable because finding it does not require a long time. There exists number of well-known heuristic techniques for solving a variety of combinatorial problems, such as: Clarke and Wright Savings algorithm, Gillett and Miller Sweep algorithm, Fisher and Jaikumar algorithm, Variable Neighborhood Search (Braysy, 2003).

Clarke & Wright (Clarke & Wright, 1964) *savings algorithm* is the best-known heuristic. Starting with an initial solution s_1, in which each customer is served by a different route, then searches for and merges two route extremities to find a feasible route. The s*weep method* by **Gillett & Miller** (Gillet & Miller, 1974), explores the customers circularly in an increasing polar angle around the depot and each customer is added to this order at the end of the current route. Renaud and Boctor (Renaud & Boctor, 2002) developed the sweep algorithm for the fleet size and mixed VRPs.

Other heuristic called *route-first cluster-second* (Beasley, 1983) method performs in two phases, first constructs a giant circuit to visit all customers and then cut this giant tour into several routes from the depot. **Fisher and Jaikumar's** (Fisher & Jaikumar, 1981) *cluster-first route-second* method, works in two steps i.e. creating feasible clusters of all the customers and then final routes are obtained by applying TSP on each cluster. **Variable Neighbourhood Search (VNS)** is a search based heuristic (Mladenovic & Hansen, 1997). The VNS approach has been successfully applied to solve variants of VRP (Braysy, 2003; Polacek, Hartl, Doerner, & Reimann, 2004).

Meta-heuristics Approaches

Meta-heuristics *(Heuristic + Randomization)* are the general algorithmic framework designed to solve complex optimization problems (Osman & Laporte, 1996). Since, few decades meta-heuristics emerging as a successful alternatives to classical heuristics for solving complex optimization problem. A wide range of meta-heuristic approaches have been introduces for solving the VRPs and this class of algorithms have two main classifications such as: simulated annealing, tabu search (Glover & Laguna,

1998), local search (Aarts & Lenstra, 2003); lies in the category of *Single Solution Approaches* of meta-heuristic class. Another is *Population-based Approaches*, which includes evolutionary algorithms (Kenneth & Jong, 2006) genetic algorithm (GA), neural network (NN), and Swarm Intelligence techniques.

Simulated Annealing (SA) (Laarhoven & Aarts, 1987) is a relaxation technique for optimization problems, which locates good approximation to global values (minimum or maximum) for the given function in the search space. Osman's implementation was a successful algorithm, which provide good solutions, but failed to compete with the tabu search algorithms proposed at that time. **Tabu Search (TS)** explores the solution space by moving at each iteration from a solution '*s*' to the best solution in a subset of its neighbourhood N(s). Tabu search approaches have been proposed widely since the past two - three decades. It is introduced by the Willard (Willard, 1989), but soon supplanted by many algorithms Osman (Osman, 1993) and Taillard (Taillard, 1993), which is one of the best-known tabu search algorithm to solve the CVRP. **Local Search (LS)** improvement heuristic (Aarts & Lenstra, 2003) starts with an initial solution, explores the neighbourhood to find a better solution to replace the existing solution, and repeats till no improving solutions can be found in the neighbourhood.

Neural networks (NN) are the computational models made up of small units interconnected to each other through some associated weights, like neurons in the brain. Artificial neural network (ANN) is a mathematical model based on biological NNs. A self-organizing map (SOM) is a type of ANN that is trained using unsupervised learning to produce a low-dimensional (typically 2-D), training samples, called as map. The pioneer work of Hopfield (Hopfield & Tank, 1986) model for solving TSP gives a start to solve the combinatorial problems. Later, the elastic net (EN) model by Durbin and Willshaw (Durbin & Willshaw, 1987), and the SOM Kohonen model (Kohonen, 1988) have been proved more efficient for solving TSP than the Hopfield model. Ghaziri (Ghaziri & Osman, 2006) have solved VRP and VRP with backhauls using NN-SOM.

Evolutionary Algorithms (EA) (Goldberg, 1989; Jong, 2006) are the part of Evolutionary Computations, based on the biological evolution and uses iterative process to find the optimal solution. The evolutionary algorithm mimics the natural evolutionary principles to drive its search towards an optimal solution. EAs uses a population of solutions rather than a single solution per iteration and the end results are also the population of solution, i.e., the Pareto-optimal set. A solution seeker can choose any solution among the solutions in the Pareto optimal set so obtained. EA's usually solves the optimization problems successfully. Homberger and Gehring (Homeberger & Gehring, 1999) described GAs to solve the VRPTW. Later a two-phase meta-heuristic is proposed by them, where in first phase GA minimizes the vehicle count, and the another phase compute the minimum total distance using tabu search. Researchers like Berger and Barkaoui (Berger & Barkaoui, 2003) developed two distinct populations evolved from GAs, pursuing different objectives, aim to minimize: (i) the total distance and (ii) the violations of the time window constraints.

Swarm Intelligence (SI) Techniques

Nature Inspired Computing (NIC) is an emerging technology, inspired by the nature's behaviour in various situations, to solve the complex problems. ***Swam Intelligence*** is a sub area of NIC and based on the study of cooperating behaviour of simple individuals (e.g., ants, bird flocking, and bees etc.) in

various decentralized systems. The population consists of simple individuals, can usually solve complex tasks by interacting locally among the individuals and with their environments. Therefore, SI is a promising way to develop powerful solutions to optimization problems.

Ant Colony Optimization (ACO) by (Dorigo, Caro, & Gambardella, 1999) is a meta-heuristic approach inspired by an analogy, that ants use pheromone trails for shortest paths when foraging for food. A moving ant marking the path with a trail of some essence called pheromone on the ground. An individual ant move randomly and when it detects a laid pheromone trail it follows that path and reinforces the trail with its own pheromone, which attracts more ants. The process is thus characterized by a positive loop, where the probability of an ant chooses a path increases with the number of ants moved on that path. Bullnheimer et. al. (B. Bullnheimer, 1997) proposed the Ant System (AS, inspired by foraging behaviour of ants) algorithm for solving the problems in VRP. Then they proposed an improved AS algorithm in 1999 and showed the competitiveness to solve the VRP in a reasonable time (Bullnheimer, Hartl, & Strauss, 1999). ACO for solving VRP is identical to the AS applied to the VRP. Therefore, the implementation of ACO with little improvement in the original algorithm follows three main steps: (i) Tour construction (ii) Route improvement strategies (iii) Pheromone updating. ACO has been successfully applied to solve various variant of VRPs as mentioned in literature, CVRP, PVRP, SDVRP, VRPPD (Montemanni, Gambardella, Rizzoli, & Donati, 2005; Kumar & Panneerselvam, 2012), VRPTW (Rizzoli, Montemanni, Lucibello, & Gambardella, 2007; Snadhaya & Katiyar, 2013). Researchers also combined ACO with other successful algorithms to solve for large-scale complex VRPs. Reimann et al. (Reimann, Doerner, & Hartl) have developed a successful approach named D-Ants (Decomposition-Ants) build on the Savings based Ant System (SbAS) and competitive to the best-known Tabu Search method in terms of solution quality and processing time.

Particle Swarm Optimization (PSO) is a population-based method proposed by Eberhart and Kennedy (Kennedy & Eberhart, 1995) and inspired by the flocking behavior of birds and fish schooling. PSO optimizes the problem by iteratively improving a population of candidate solutions called particles, moving around the large search-space; adjusting its position and velocity according to the neighbouring particles. Each particle's movement is influenced by its local best known position *(lbest)* and local best known velocity (*pbest)* and also guided by the global best *(gbest)* known position and velocity of its neighbors in the search-space. This way, while keep updating for better positions within the neighborhood the swarm moves toward the best optimal solutions. PSO has been successfully applied in many research and application areas and achieved better results in a faster, cheaper way as compared with other methods. Kachitvichyanukul (Ai & Kachitvichyanukul, 2009) developed a PSO algorithm for VRPPD and compared the performance of their method with other existing meta-heuristics on some benchmark problems. They also (Ai & Kachitvichyanukul, 2009) used a similar PSO for the CVRP. PSO has been also applied to other logistics problems (Ai & Kachitvichyanukul, 2009), (Ai & Kachitvichyanukul, 2008).

Bees Algorithm is introduced by Karaboga Dervis (Karaboga, 2005) and later (Karaboga & Akay, 2011) modified it as a new meta-heuristic approach. This algorithm is inspired from the intelligent behaviour of bees searching for food (nector). A bee's colony is made up of a ruler (queen) with numerous male drones and a large number of female labour (non-reproductive). The job of the queen is to mate with the drones and to start new colonies. The worker bees construct the honeycomb. The duty of the young bees is to give food to the queen and to the male drones, clean and protect the entire

colony and to accumulate more food. The characterized interacting collective behaviour of specialized individuals, distributed simultaneous tasks performed, and self-organization, leads to the colony as an organized teamwork system. The Artificial Bees Colony (ABC) is an iterative algorithm, where each *agent* (bee) is assigned to a solution (food source) which is generated randomly; then in every iteration, each agent uses neighbourhood operator to find a new solution. The *fitness function (*nectar amount) of the newly found solution is evaluated per iteration; if it is found higher then replace the previous solution by the newer one. Gomez et.al. (Gomez & Salhi, 2014) presented a new ABC algorithm for solving the CVRP. Y. Marinakis et.al. (Marinakis, Mariniki, & Dounias, 2008) proposed honey bees mating algorithm for VRP named it as HBMOVRP, which successfully solves the VRPs.

Intelligent Water Drops (IWD) is introduced by (Shah-Hosseini, 2007) is a constructive algorithm, inspired from natural water drops that flow in the rivers, that how they divert to find an optimal path to their destination. IWDs have two main properties:

(i) Velocity (certain initial velocity): Less soil more velocity gains in IWD and vice-versa.

(ii) Soil (initially zero): Soil to an IWD is added from location to location.

In IWD algorithm, the artificial water drops change their environment to choose optimal path with less soil on its links i.e. also for the next location to move paths should have less soil. The solutions are therefore incrementally formulated by the IWD algorithm. Shah-Hosseini tested a modified IWD algorithm on travelling salesman problem and obtained near-optimal solutions. Rayapudi (Rayapudi, 2011) also implemented IWD algorithm for solving economic load dispatch problem.

Bacterial Foraging Optimization Algorithm (BFOA), proposed by Passino (Niu, Wang, Tan, & Jing-Wen Wang, 2012), is inspired by the social foraging behavior of *Escherichia coli* (*E. coli*) bacteria. It has been largely accepted by many practionars for solving real-world optimization problems in several application domains. Key idea of BFOA is to mimic the chemo-taxis movement (bacterium searching for nutrients in small moves) of virtual bacteria in the problem search space. When these E.coli bacteria get the sufficient food, they are increased in length and break in the middle to make clones. Each individual can communicate with others by sending signals. Ben Niu et.al. develops a novel BFO with adaptive chemo-taxis step to solve VRPTW. Lijing Tan et.al (Tan, Lin, & Wang, 2015) proposes a BFO variant i.e. adaptive comprehensive learning bacterial foraging optimization (ALCBFO) to solve VRPTW which shows significantly better performance in solving multimodal problems.

Gravitational Search Algorithm (GSA), a new algorithm is introduced by Rashedi, E. (Liu, Qiu, & Zhan, 2014) which is formulate on the basis of laws of gravity and mass interaction. In this algorithm interaction between agents (collection of masses) takes place on the basis of Newtonian's law of gravity and motion. Every agent try to attract others by this gravitational force, which cause a motion in all the agents towards the objects of heavy masses. The objects that are heavier in masses are considered to good solutions. By lapse of time, the objects (masses) that are attracted by heavy mass are representing optimal solution in the search space. They also proposes an improved GSA (IGSA) for solving VRP.

River Formation Dynamics (RFD) (Rabanal, Rodríguez, & Rubio, 2007) is a new algorithm, based on mimicking how water forms the rivers by wear away the land and settling the remains. The water drops alter the land by changing i.e. increasing or decreasing the altitude of places and the paths with decreasing altitudes gives the best solutions. Decreasing banks of river (gradients) are constructed and

are followed by upcoming water drops. These drops then compose new gradients and reinforces (strengthen) the best ones. They applied RFD to solve some NP-complete problems: (i) TSP (ii) minimum spanning tree (MSV) problem and compared the results obtained in these problems with the ACO results.

1.5 Parallel Computation

In parallel computation, large and complex problems are divided into many small problems in order to reduce the computation time by solving them concurrently. Usually, an algorithm is written and implemented as a stream of serial instructions which are executed one by one on a computer, which takes very long time. Here the parallel computing techniques are important which allows multiple processes to run simultaneously, as problem is divided into smaller ones and distributed to several processors. Parallel computers can be classified as multi-core and multi-processor computers that can execute multiple processes on a single machine, while clusters, grids, and MPPs use different computers for the same computation. Parallel computing programs are more difficult to write than sequential programs (Jozefowiez, Semet, & Talbi, 2002) because concurrent computation introduces new software bugs, like race condition which is most common. Also, synchronization and communication between subtasks is one of the biggest obstacle for a good parallel program.

2. Literature Review

Extensive literature survey has done to understand the vehicle routing problems, the existing methods that have been applied to solve VRPs in a better and efficient way. It is found in past studies that the exact methods such as Branch and Bound, Branch-Cut-and-Price, Set-Partitioning and Column Generation, have a size limit and failed to deal when the dimension and the complexity of the problem increases (Gambardella, et al., 2003). Therefore, to overcome this problem the literature offers a wealth of heuristic and meta-heuristic approaches to solve the VRPs. The findings are discussed below:

Dharmapriya et.al. (Dharmapriya, Siyambalapitiya, & Kulatunga, 2010) proposed the applications of artificial intelligence (AI) techniques such as tabu search and simulated annealing to VRP. In their work they extended the general VRP to Multi Depot VRP with Time Windows and Split Delivery (MDVRPTWSD). In another work (Goel & Gruhn, 2005; Gandreau, Guertin, Potvin, & Seguin, 2006), they addresses the several contarints like capacity, heterogenous vehicles with different travel times, travel costs, time windows constraints, multiple locations for pickup and delivery and different starting and ending points for vehicles. In their algorithm an iterative improvement based on local neighbourhood search for instances 50, 100, 250 and 500 is used. A reduced Variable Neighbourhood Search algorithm is used for swapping the elements between neighbourhoods. This type of combined helps to avoid traaping in local minima.

Nazif and Lee (Nazif & Lee, 2010) proposed a GA with an optimized crossover operator designed by a complete undirected graph. This operator can find an optimal set of delivery routes while satisfying the standard requirements and gives minimum total cost to solve VRPTWs. The proposed approach introduced various techniques for better enhancement of solutions quality. Najera (Najera, 2010) proposed Multi-Objective Evolutionary Algorithm to solve two variants of the bi-objective Vehicle Routing Problem and obtained good results. A hybrid two-phase approach of set partitioning method (formulate VRP) with genetic algorithm (for natural reproduction, selection and evolution) is proposed

(Alvarenga, Mateus, & Tomi, 2007) to solve the VRPTW. The tests are produced using real numbers and truncated data type, and results are compared against previously proposed heuristic and exact methods, which show that the proposed approach outperforms all published heuristic methods in terms of the minimal travel distance. Wilck and Cavalier (Wilck & Cavalier, 2012) proposed two hybrid GAs, both with dissimilar fitness approach, to solve SDVRP for 32 data sets. Shen Y. and Murata (Shen & Murata, 2012) presented a GA for the VRP with 2-D loading constraints (2L-CVRP), i.e. combining the BPP (Bin Packing Problem) and the VRP.

Gambardella et.al. (Gambardella, Taillard, & Agazzi, 1999) proposed ACO based algorithm to solve VRP with time windows, and called it as Multiple Ant Colony System for VRPTW (MACS-VRPTW). In their approach they designed the artificial colonies to efficiently optimize the multiple objective functions; such as the first colony minimizes the vehicle count while the second minimizes the total distance travelled. The result obtained show that their algorithm is competitive with the best known algorithms. Sandhaya et.al. (Snadhaya & Katiyar, 2013) presented an enhanced Ant Colony System (ACS) to solve VRPTW problem efficiently. Experiments are performed with the Solomon data sheet (benchmark for VRPs) and the results obtained are compared with the best known results available in literature. Bell, J. E. (Bell & McMullen, 2004) solved VRP using ACO and shows that ACO is successful in finding solutions near the best-known solutions for problems with up to 20 demand locations. They also found ACO to superior than savings algorithm to apply on large, complex logistics-oriented VRPs.

Manfrin (Manfrin, 2004) applied ant colony optimization on the vehicle routing problem and compared the results of five meta-heuristics namely ACO, Evolutionary Computation, Tabu Search, Iterated Local Search, and Simulated Annealing. Rizzoli et al. (Rizzoli, Oliverio, Montemanni, & Gambardella, 2004) applied ACO to various instances of VRP such as VRP with time windows, Dynamic VRP, VRP with Pickup and Delivery. They also applied the proposed algorithm to two real-world, industrial-scale applications. Bin, Zhong-Zhen applied ACO to solve VRP (Bin, Zhong-Zhen, & Baozhen, 2009). They possessed a new technique to update the trails of the pheromone and names it as ant-weight strategy, and also used GA (mutation operator) on it.

Bouhafs (Bouhafs, Hajjam, & Koukam, 2010) combined ant colony system algorithm with a savings algorithm and applied a local search heuristic to get better results for the CVRP. Saravanan (Saravanan & Sundararama, 2010) proposed an approach based on ACO to one-sided time constraint VRP, where the delivery of products (from the depot) to the distribution centres has to completed within the maximum permissible time. They compared their proposed approach with the shortest path based heuristic approach. They found that their ACO-based approach is more efficient in solving the considered VRP in comparison to the heuristic approach.

PSO has been widely used in many research and application domains and achieved good results in a fast and cheaper way as compared to others. Kachitvichyanukul (Ai & Kachitvichyanukul, 2009) developed a PSO algorithm for VRPPD and compared its performance with other existing meta-heuristics on some benchmark problems. They also used a similar PSO for the CVRP, VRPTW. Some other researchers modified PSO and applied it to VRP to get better results. Kanthavel (Kanthavel &

Prasad, 2011), proposed a new version of PSO, named Nested PSO for solving VRP, they focused on maximum utilization of loading capacity and determining the optimum set of vehicle routes for CVRP. Some hybrid algorithms have been also proposed by many researchers. Yin and Liu (Yin & Liu, 2009) hybridized PSO with GA (crossover) and proposed an improved GPSO algorithm to solve a single-depot complex VRP (SCVRP). According to them this combination is able to avoid being trapped in local optimum. Ponce (Ponce) proposed a PSO based algorithm for solving VRP. In this approach each particle represents a set of feasible vehicle routes, after the construction of initial routes, a modified edge recombination crossover operator (GA) is used to move particles towards better solutions. Bin, et.al. Applied ACO to solve VRP (Bin, Zhong-Zhen, & Baozhen, 2009), they possessed a new technique named ant-weight strategy to update pheromone value and used mutation operator (GA) to explore the solution space. Ai-ling (Ai-ling, Gen-ke, & Zhi-ming, 2006) proposed a hybrid algorithm to solve the CVRP. In the proposed algorithm, the authors hybridized Discrete PSO (DPSO) with simulated annealing. In this approach DPSO search for the optimal result globally and locally in the search space and SA is used to avoid the convergence to a local optima. The results obtained in the study shows that the proposed hybrid approach is a feasible and effective approach for CVRP, especially for large problems.

Y. Marinakis et.al. (Marinakis, Mariniki, & Dounias, 2008) proposed honey bees mating algorithm, for efficiently solving the VRP. The proposed HBMOVRP algorithm combined with the Multiple Phase Neighbourhood Search and Greedy Randomized Adaptive Search Process (MPNS-GRASP) algorithm. Gomez et.al. (Gomez & Salhi, 2014) proposed a new artificial bee's colony algorithm for solving the CVRP. The novel approach of the algorithm relies upon the two specializations named diversification and intensification. Two commonly used benchmark datasets are used to test the performance of the algorithm. Kamkar et al. (Kamkar, Akbarzadeh-T, & Y aghoobi, 2010) applied IWD algorithm to VRP and solves by modelling the behaviour of water drops of collectively modify the environment and optimize their path.

Ben Niu et.al. (Niu, Wang, Tan, & Jing-Wen Wang, 2012) developed a novel BFOA algorithm with adaptive chemotaxis step to solve VRPTW. The proposed algorithm is proved more efficient when compared with other BFO algorithms, and confirms its efficiency to solve VRPTW. Tan et.al (Tan, Lin, & Wang, 2015) proposes an adaptive comprehensive learning BFO optimization (ALCBFO). Later, the results are compared with the classical GA, PSO, the original BFO, ACLBFO shows significantly better in solving multimodal problems. Liu. N. (Liu, Qiu, & Zhan, 2014) proposes an improved GSA (IGSA) combined with the GA (cross operator). Finally, the 50 and 100 group instances of VRPMTW are solved by GSA and IGSA separately. The result shows, IGSA is much better than GSA in various aspects such as: the optimal path length, average delay time, etc.

Many researchers also explore the possibilities of parallel computation on different algorithms to solve large VRPs. Groer et.al. (Groer, Golden, & Wasil, 2010) proposed parallelization technique to solve VRP by PSO. The outcome results by their algorithm are quick and better in comparison with the results obtained by Genetic Algorithms. Doerner et.al. (Doerner, Hartl, & Lucka) proposed a parallel implementation on the D-Ant algorithm; originally developed by (Reimann, Doerner, & Hartl) to solve the VRP. They used the ants to solve only sub-problems rather than solving the whole problem; to

speed up the operation. Czech & Czarnas (Czech & Czarnas) proposed simulated annealing algorithm with parallel computation to solve the VRPTW.

3. Motivation

Since VRP is an *NP-hard* problem, which means no exact and efficient method can give the optimal solution to the problem in polynomial time. VRP has many additional objectives to be optimised and several constraints to be considered, leads to different versions of VRPs that have been introduced in the last few decades. Various approximation methods have already proposed to solve the VRP by a number of researchers for decades. But several methods have limitations when dimensions (number of clients or orders) and complexity (number of constraints, vehicles or depots) of the problem increases (Gamberdalla, et al.). To overcome these problems, researchers proposed Swarm Intelligence (SI) based techniques to solve VRP, which gives near optimal solutions to the problem in comparison to other existing methods. Since then SI techniques emerged as successful approaches to solve large and complex optimization problem (like VRP) and also become an important research area to work on. Therefore, motivated by both i.e. VRP's practical importance and considerable difficulty and the advantageous features of SI techniques to solve the complex optimization problems, the proposed research problem has been framed. In particular the motivation is to implement the SI techniques to solve certain variants of the vehicle routing problem in an efficient manner.

4. Objective

It is proposed to:

1. Study how various Swarm Intelligence (SI) techniques can be used to solve variants of VRPs such as Capacitated VRP with Time Windows, Pickup and Delivery etc.
2. Explore the possibility of parallel implementation of SI techniques to solve these variants of VRP.
3. Implementation of the newly developed techniques and algorithms in SI on the variants of VRPs under study.
4. Comparison of the newly developed techniques with existing techniques on standard benchmark problems.

5. Platform

MATLAB will be used as the main tool to run simulations, and to compute and analyse the results of the newly developed algorithms.

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